Dynamics of price and trading volume in a spin model of stock markets with heterogeneous agents

Taisei Kaizoji [a,1], Stefan Bornholdt [b] Yoshi Fujiwara [c]

[a] *Division of Social Sciences, International Christian University, Osawa, Mitaka, Tokyo 181-8585, Japan*

[b] *Institute for Theoretical Physics, University of Kiel, Leibnizstrasse 15, D-24098 Kiel, Germany*

[c] *Keihanna Research Center, Communications Research Laboratory, Kyoto 619-0289, Japan*

Abstract

The dynamics of a stock market with heterogeneous agents is discussed in the framework of a recently proposed spin model for the emergence of bubbles and crashes. We relate the log returns of stock prices to magnetization in the model and find that it is closely related to trading volume as observed in real markets. The cumulative distribution of log returns exhibits scaling with exponents steeper than 2 and scaling is observed in the distribution of transition times between bull and bear markets.

Key words: Econophysics, Stock Market, Spin Model, Volatility
PACS: 89.90.+n, 02.50.-r, 05.40.+j

1 Introduction

During the last years there has been great interest in applications of statistical physics to financial market dynamics. A variety of agent-based models have been proposed over the last few years to study financial market dynamics [1–5]. In particular, spin models as the most popular models of statistical mechanics

Email address: `kaizoji@icu.ac.jp` (Taisei Kaizoji).
[1] Corresponding author. Division of Social Sciences, International Christian University, Osawa, Mitaka, Tokyo 181-8585, Japan. FAX: +81-422-33-3367.

have been applied to describe the dynamics of traders in financial markets by several researchers [6–12].

A particularly simple model of a stock market in the form of a spin model motivated by the Ising model has been proposed recently [6], in order to simulate the dynamics of expectations in systems of many agents. The model introduces a new coupling that relates each spin (agent) to the global magnetization of the spin model, in addition to the ferromagnetic (Ising) couplings connecting each spin to its local neighborhood. The global coupling effectively destabilizes local spin orientation, depending on the size of magnetization. The resulting frustration between seeking ferromagnetic order locally, but escaping ferromagnetic order globally, causes a metastable dynamics with intermittency and phases of chaotic dynamics. In particular, this occurs at temperatures below the critical temperature of the Ising model. While the model exhibits dynamical properties which are similar to the stylized facts observed in financial markets, a careful interpretation in terms of financial markets is still lacking. This is the main goal of this paper. In particular, [6,12] treat the magnetization of the model as price signal which, however, is unnatural when deriving a logarithmic return of this quantity as these authors do. As a result, small magnetization values cause large signals in the returns with an exponent of the size distribution different from the underlying model's exponent.

Let us here consider Bornholdt's spin model in the context of a stock market with heterogeneous traders. The aim of this paper is (i) to interpret the magnetization of the spin model in terms of financial markets and to study the mechanisms that create bubbles and crashes, and (ii) to investigate the statistical properties of market price and trading volume. The new elements in the model studied in this paper are the explicit introduction of two groups of traders with different investment strategies, fundamentalists and interacting traders [2], as well as of a market clearing system that executes trading at matched book. Given these conditions, the market price is related to the sum of fundamental price and magnetization, and the trading volume is simply given by the magnetization. We also show that the model is able to explain the empirically observed positive cross-correlation between volatility and trading volume [3]. Finally, we observe that the model reproduces the well-known stylized facts of the return distribution such as fat tails and volatility clustering [15,16], and study volatilities at different time-scales.

[2] The interacting traders are often called noise traders in finance literature.

[3] The positive cross-correlation between volatility and trading volume is demonstrated by [13]. However, little attention has been paid to the relationship between price and trading volume in terms of theory, Iori [14] being among those who first studied this relationship.

2 The Model

Let us consider a stock market where a large stock is traded at price $p(t)$. Two groups of traders with different trading strategies, *fundamentalists* and *interacting traders*, participate in the trading. The number of fundamentalists m and the number of interacting traders n are assumed to be constant. The model is designed to describe the stock price movements over short periods, such as one day. In the following, a more precise account of the decision making of each trader type is given.

2.1 Fundamentalists

Fundamentalists are assumed to have a reasonable knowledge of the fundamental value of the stock $p^*(t)$. If the price $p(t)$ is below the fundamental value $p^*(t)$, a fundamentalist tends to buy the stock (as he estimates the stock to be undervalued), and if the price is above the fundamental value, a fundamentalist tends to sell the stock (then considered as an overvalued and risky asset). Hence we assume that fundamentalists' buying or selling order is given by:

$$x^F(t) = a\, m\; (\ln p^*(t) - \ln p(t))\,, \tag{1}$$

where m is the number of fundamentalists, and a parametrizes the strength of the reaction on the discrepancy between the fundamental price and the market price.

2.2 Interacting Traders

During each time period, an interacting trader may choose to either buy or sell the stock, and is assumed to trade a fixed amount of the stock b in a period of trading. Interacting traders are labeled by an integer $1 \leq i \leq n$. The investment attitude of interacting trader i is represented by the random variable s_i and is defined as follows: If interacting trader i is a buyer of the stock during a given period, then $s_i = +1$, otherwise he sells the stock and $s_i = -1$.

Now let us formulate the dynamics of the investment attitude of interacting traders in terms of the spin model [6]. Let us consider that the investment attitude s_i of interacting trader i is updated with a heat-bath dynamics according to

$$s_i(t+1) = +1 \quad \text{with} \quad p = \frac{1}{1+\exp(-2\beta h_i(t))} \tag{2}$$

$$s_i(t+1) = -1 \quad \text{with} \quad 1-p \tag{3}$$

where $h_i(t)$ is the local field of the spin model, governing the strategic choice of the trader.

Let us consider the simplest possible scenario for local strategy changes of an interacting trader. We assume that the decision which an interacting trader makes is influenced by two factors, local information, as well as global information. Local information is provided by the nearby interacting traders' behavior. To be definite, let us assume that each interacting trader may only be influenced by its nearest neighbors in a suitably defined neighborhood. Global information, on the other hand, is provided by the information whether the trader belongs to the majority group or to the minority group of sellers or buyers at a given time period, and how large these groups are. The asymmetry in size of majority versus minority groups can be measured by the absolute value of the magnetization $|M(t)|$, where

$$M(t) = \frac{1}{n}\sum_{i=1}^{n} s_i(t). \tag{4}$$

The goal of the interacting traders is to obtain capital gain through trading. They know that it is necessary to be in the majority group in order to gain capital, however, this is not sufficient as, in addition, the majority group has to expand over the next trading period. On the other hand, an interacting trader in the majority group would expect that the larger the value of $|M(t)|$ is, the more difficult a further increase in size of the majority group would be. Therefore, interacting traders in the majority group tend to switch to the minority group in order to avert capital loss, e.g. to escape a large crash, as the size of the majority group increases. In other words, the interacting trader who is in the majority group tends to be a risk averter as the majority group increases. On the other hand, an interacting agent who is in the minority group tends to switch to the majority group in order to gain capital. An interacting agent in the minority group tends to be a risk taker as the majority group increases.

To sum up, the larger $|M(t)|$ is, the larger the probability with which interacting traders in the majority group (interacting traders in the minority group, respectively) withdraw from their coalition. Following [6], the local field $h_i(t)$ containing the interactions discussed above is specified by

$$h_i(t) = \sum_{j=1}^{m} J_{ij} S_j(t) - \alpha S_j(t)|M(t)| \tag{5}$$

with a global coupling constant $\alpha > 0$. The first term is chosen as a local Ising Hamiltonian with nearest neighbor interactions $J_{ij} = J$ and $J_{ii} = 0$ for all other pairs.

We assume that the interacting-traders' excess demand for the stock is approximated as

$$x^I(t) = b\, n\, M(t). \tag{6}$$

2.3 Market price and trading volume

Let us leave the traders' decision-making processes and turn to the determination of the market price. We assume the existence of a market clearing system. In the system a *market maker* mediates the trading and adjusts the market price to the market clearing values. The market transaction is executed when the buying orders are equal to the selling orders.

The balance of demand and supply is written as

$$x^F(t) + x^I(t) = a\, m\, [\ln p^*(t) - \ln p(t)] + b\, n\, M(t) = 0. \tag{7}$$

Hence the market price and the trading volume are calculated as

$$\ln p(t) = \ln p^*(t) + \lambda M(t), \quad \lambda = \frac{b\, n}{a\, m}, \tag{8}$$

and

$$V(t) = b\, n\, \frac{1 + |M(t)|}{2}. \tag{9}$$

Using the price equation (8) we can categorize the market situations as follows: If $M(t) = 0$, the market price $p(t)$ is equal to the fundamental price $p^*(t)$. If $M(t) > 0$, the market price $p(t)$ exceeds the fundamental price $p^*(t)$ (*bull* market regime). If $M(t) < 0$, the market price $p(t)$ is less than the fundamental price $p^*(t)$ (*bear* market regime). Using (8), the logarithmic relative change of price, the so-called log-return, is defined as

$$\ln p(t) - \ln p(t-1) = (\ln p^*(t) - \ln p^*(t-1)) + \lambda (M(t) - M(t-1)). \tag{10}$$

Let us consider for a moment that only fundamentalists participate in trading. Then in principle the market price $p(t)$ is always equal to the fundamental price $p^*(t)$. This implies that the so-called *efficient market hypothesis* holds. Following the efficient market model by [17] then the fundamental

price $p^*(t)$ follows a random walk. Since the continuous limit of a random walk is a Gaussian process, the probability density of the log-return, defined as $r(t) = \ln p(t) - \ln p(t-1)$, is normal. For real financial data, however, there are strong deviations from normality [4]. As we discuss here, including both fundamentalists and interacting traders to coexist in the market, offers a possible mechanism for excessive fluctuations such as bull markets and bear markets.

To investigate the statistical properties of the price and the trading volume in the spin model of stock markets, we will assume for simplicity that the fundamental price is constant over time.

3 Simulations

3.1 Bubbles and crashes

In the new framework developed so far we see that the dynamics of the log-return corresponds to the linear change in absolute magnetization of the spin model [6]. Typical behavior of the such defined return $r(t)$ as well as the magnetization $M(t)$ is shown in Figures 1(a) and 1(b). Here, a 101*101 lattice of the general version of the model as defined in eq. (8) and with the spins updated according to (5) is shown. It is simulated at temperature $T = 1/\beta = 0.5$ with couplings $J = 1$ and $\alpha = 20$, using random serial and asynchronous heat bath updates of single sites. In Figure 1(a) the intermittent phases of ordered and turbulent dynamics of the log-return are nicely visible. Qualitatively, this dynamical behavior is similar to the dynamics of daily changes of real financial indices, as for example the Dow Jones Index shown in Figure 1(c). To some degree, these transitions of the return can be related to the magnetization in the spin model. Figure 1 (b) illustrates that the bull (bear) market $M(t) > 0$ ($M(t) < 0$) becomes unstable, and the transition from a metastable phase to a phase of high volatility occurs, when the absolute magnetization $|M(t)|$ approaches some critical value. Noting that trading volume is defined as $b\, n\, (1 + |M(t)|)/2$, this suggests that also some critical trading volume exists near the onset of turbulent phases. This is in agreement with the empirical study by [13] who found the empirical regularities: (i) positive correlation between the volatility and the trading volume; (ii) large price movements are followed by high trading volume.

The origin of the intermittency can be seen in the local field h_i eq. (5) rep-

[4] Furthermore financial asset prices are too volatile to accord with efficient markets as has been demonstrated by [18] and [19].

resenting the external influences on the decision-making of interacting-trader i. In particular, the second term of h_i tends to encourage a spin flip when magnetization gets large. Thus each interacting-trader frequently switches his strategy to the opposite value if the trading volume gets large. As a consequence, the bull (bear) market is unstable and the phase of the high volatility appears. The metastable phases are the analogue of speculative bubbles as, for example, the bull market is defined as a large deviation of the market price from the fundamental price. In fact it is a common saying that "it takes trading volume to move stock price" in the real stock market. A typical example is the crash of Oct. 1987, when the Dow Jones Industrial Average dropped 22.6% accompanied by an estimated 6×10^8 shares that changed hands at the New York Stock Exchange alone [20]. Fama [21] has argued that the crash of Oct. 1987 at the US and other stock markets worldwide could be seen as the signature of an efficient reassessment of and convergence to the correct fundamental price after the long speculative bubble proceeding it.

It is interesting to investigate how long the bull or bear markets last from the point of view of practical use. Figure 2 shows the distribution of the bull (bear) market durations that is defined as the period from the beginning of a bull (bear) market $M(t) \geq 0$ ($M(t) \leq 0$) to the end of the bull (bear) market $M(t) = 0$. In other words, the bull (bear) market duration means the period from a point of time that the market price exceeds the fundamental price to the next point of time that the market price falls short of the fundamental price. In the model one observes that the bull (or bear) market durations are power law distributed with an exponent of approximately -1.3.

3.2 Fat tails and volatility clustering

As shown in the previous works [6] and [12] the simple spin model considered here reproduces major stylized facts of real financial data. The actual distribution of log-return $r(t)$ has *fat tails* in sharp contrast to a Gaussian distribution (Figure 3). That is, there is a higher probability for extreme values to occur as compared to the case of a Gaussian distribution. Recent studies of the distribution for the absolute returns $|r(t)|$ report power law asymptotic behavior,

$$P(|r(t)| > x) \sim \frac{1}{x^{\mu}} \tag{11}$$

with an exponent μ between about 2 and 4 for stock returns. Figure 4 shows the cumulative probability distribution of the absolute return that is generated from the model. The observed model exponent of $\mu = 2.3$ lies in the range of empirical data.

Numerous empirical studies show that the volatility $|r(t)|$ on successive days is positively correlated, and these correlations remain positive for weeks or months. This is called clustered volatility. Furthermore the autocorrelation function for volatility decays slowly, and sometimes a power law decay is observed. As seen in Figure 1(a), phases of high volatility in the model dynamics are strongly clustered. The corresponding autocorrelation of volatility $|r(t)|$ is shown in Figure 5, with volatility clustering on a qualitatively similar scale as observed in real financial market data.

3.3 Volatilities at different time-scales

Let us consider a time-scale τ at which we observe price fluctuations. The log-return for duration τ is then defined as $r_\tau(t) = \ln(P(t)/P(t-\tau))$. Volatility clustering as described in the previous section is this observable defined for an interval τ ranging from several minutes to more than a month or even longer. In this intermediate and strongly correlated regime of time-scales, volatilities at different time-scales may show self-similarity [22–25].

Self-similarity in this context states that volatilities $v_\tau \equiv |r_\tau|$ at different time-scales τ are related in such a way that that the ratio of volatilities at two different scales does not statistically depend on the coarse-graining level. Thus daily volatility is related to weekly, monthly volatilities by stochastic multiplicative factors. The self-similarity has been shown to be equivalent to scaling of moments under some conditions [22][23][24]. Scaling occurs when $\langle \, {v_\tau}^q \, \rangle \propto t^{\phi(q)}$, where $\phi(q)$ is the scaling function which is related to the statistical property of the multiplicative factors [25]. Figure 6 (a) depicts the presence of self-similarity in actual data of the NYSE stock index.

Though it is not straightforward to relate time-scales between the simulation and real data, it is interesting to look at how the volatilities at different time-scales behave in the regime where the volatility clustering is valid. Figure 6 (b) shows the scaling of moments from the data of log-returns calculated for different scales in time-steps of the simulation. We observe a range where self-similarity dominates and that this property is broken at some time-scale, which corresponds to the scale where volatility clustering as seen in Figure 5 deviates from a power-law of the autocorrelation function. This observation is encouraging as it might help relate the time-scales of simulations and real markets.

4 Concluding remarks

In this paper we have considered the spin model presented in [6] in the context of a stock market with heterogeneous traders, that is, fundamentalists and interacting traders. We have demonstrated that magnetization in the spin model closely corresponds to trading volume in the stock market, and the market price is determined by magnetization under natural assumptions. As a consequence we have been able to give a reasonable interpretation to an aperiodic switching between bull markets and bear markets observed in Bornholdt's spin model. As a result, the model reproduces main observations of real financial markets as power-law distributed returns, clustered volatility, positive cross-correlation between volatility and trading volume, as well as self-similarity in volatilities at different time-scales. We also have found that scaling is observed in the distribution of transition times between bull and bear markets. Although the power law scaling of the distribution has never been examined empirically on short time scales, the power law statistics showed here is not only an interesting finding theoretically but presumably also useful to measure the risk of security investments in practice. The empirical study will be left for future work.

References

[1] P. Bak, M. Paczuski, and M. Shubik, Price variation in a Stock Market with Many Agents, Physica A **246**, 430 (1997).

[2] D. Challet and Y.-C. Zhang, Emergence of Cooperation and Organization in an Evolutionary Game, Physica A **246** 407 (1997).

[3] R. Cont and J.P. Bouchaud, Macroeconomic Dynamics **4**, 170 (2000).

[4] T. Lux and M. Marchesi, Scaling and Criticality in a Stochastic Multi-Agent Model of a Financial Market, Nature **397** 498 (1999).

[5] D. Stauffer and D. Sornette, Self-organized percolation model for stock market fluctuations, Physica A **271** 496 (1999).

[6] S. Bornholdt, Expectation bubbles in a spin model of markets: Intermittency from frustration across scales, Int. J. Mod. Phys. C **12** 667 (2001).

[7] D. Chowdhury and D. Stauffer, A generalized spin model of financial markets, Eur. Phys. J. B **8** 477 (1999).

[8] G. Iori, Avalanche dynamics and trading friction effects on stock market returns, Int. Mod. Phys. C **10** 1149 (1999).

[9] T. Kaizoji, Speculative bubbles and crashes in stock markets: An interacting-agent model of speculative activity, Physica A **287** 493 (2000).

[10] B.M. Roehner and D. Sornette, Thermometers of speculative frenzy, Eur. Phys. J. B **16** 729 (2000).

[11] K. Sznajd-Weron and R. Weron, A simple model of price formation, Int. J. Mod. Phys. C **13** 115 (2002).

[12] T. Yamano, Bornholdt's spin model of a market dynamics in high dimensions, Int. J. Mod. Phys. C **13** 89 (2002).

[13] A.R. Gallant, P.E. Rossi, and G. Tauchen, Stock prices and volume, Review of Financial Studies **5** 2, 199 (1992).

[14] G. Iori, A microsimulation of traders activity in the stock market: The role of heterogeneity, agents' interactions and trade frictions, www.arxiv.org/abs/adap-org/9905005 (2001); Journal of Economic Behavior and Organization, Vol. 49, no. 1 to appear in 2002.

[15] P. Gopikrishnan, V. Plerou, Y. Liu, L.A.N. Amaral, X. Gabaix, and H.E. Stanley, Scaling and correlation in financial time series, Physica A **287** 362 (2000).

[16] Y. Liu, P. Gopikrishnan, P. Cizeau, M. Meyer, C-K. Peng, and H.E. Stanley, The statistical properties of the volatility of price fluctuations, Phys. Rev. E **60** 1390 (1999).

[17] E. F. Fama, Efficient capital market: a review of theory and empirical work, Journal of Finance **25** 383 (1970).

[18] S. LeRoy and R. Poter, The present value relation, Econometrica **49** 555 (1981).

[19] R. Shiller, Do stock prices move too much to be justified by subsequent changes in dividends?, American Economic Review **71** 421 (1981).

[20] P. Gopikrishnan, V. Plerou, X. Gabaix, and H.E. Stanley, Statistical properties of share volume traded in financial markets, Phys. Rev. E. (Rapid Comm.), **62** R4493 (2000).

[21] E.F. Fama, Perspectives on October 1987, or, what did we learn from the crash?, in R. Barro, et. al., Black Monday and the Future of Financial Markets, Dow Jones-Irwin, Inc., Homewood, Illinois. 71 (1989).

[22] B. B. Mandelbrot, A. Fisher, and L. Calvet, Cowles Foundation Discussion Papers 1164, 1165, 1166, Yale University (1997).

[23] U. A. Müller et al., "Volatilities of different time resolutions – analyzing the dynamics of market components", J. Empirical Finance **4** 213 (1997).

[24] A. Arneodo, J.-F. Muzy, and D. Sornette, "Direct causal cascade in the stock market", Eur. Phys. J. B **2** 277 (1998).

[25] Y. Fujiwara and H. Fujisaka, "Coarse-graining and self-similarity of price fluctuations", Physica **A294** 439 (2001).

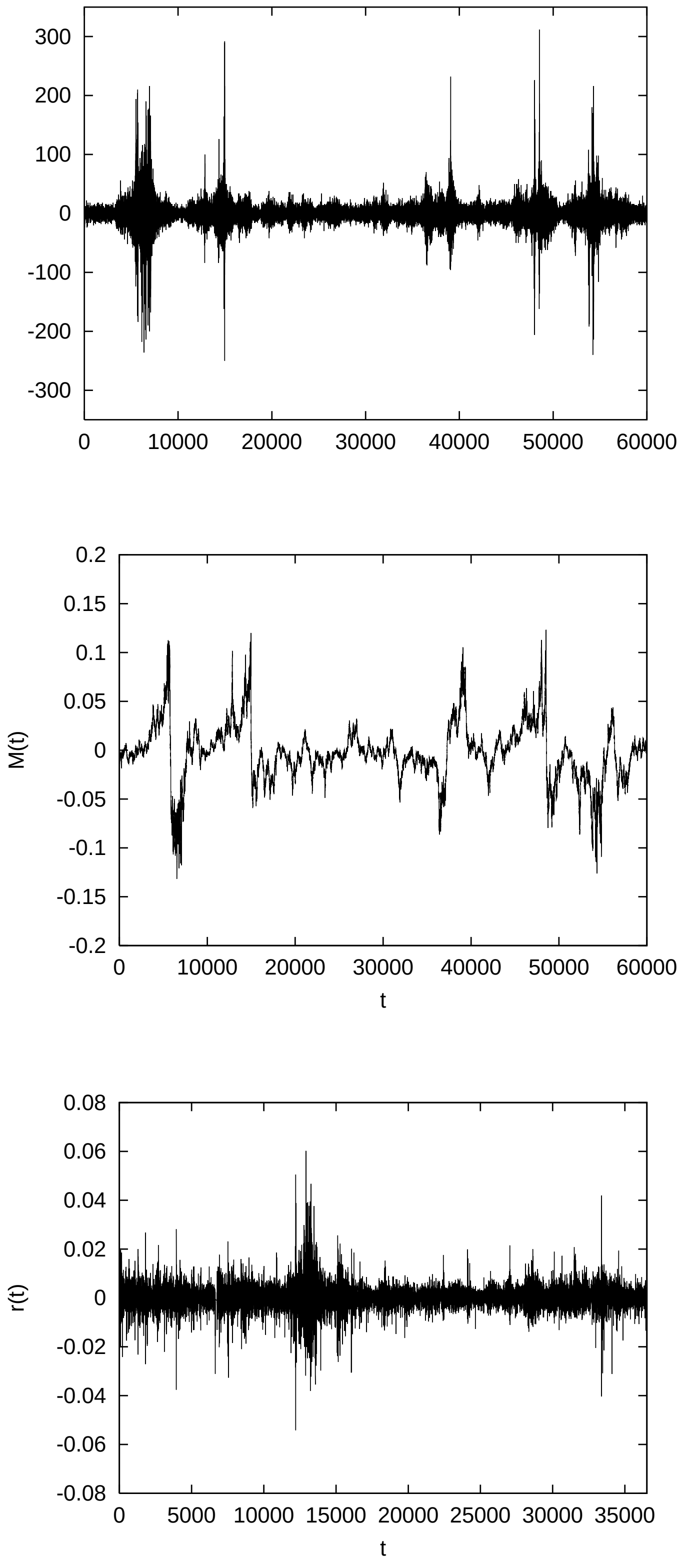


Fig. 1. (a) Logarithmic return $r(t) = \ln p(t) - \ln p(t-1)$, defined as change in magnetization $M(t)$ in the spin model. (b) Magnetization $M(t)$ of the spin model. (c) For comparison with (a), the log-return of the Dow Jones daily changes 1896-1996 is shown.

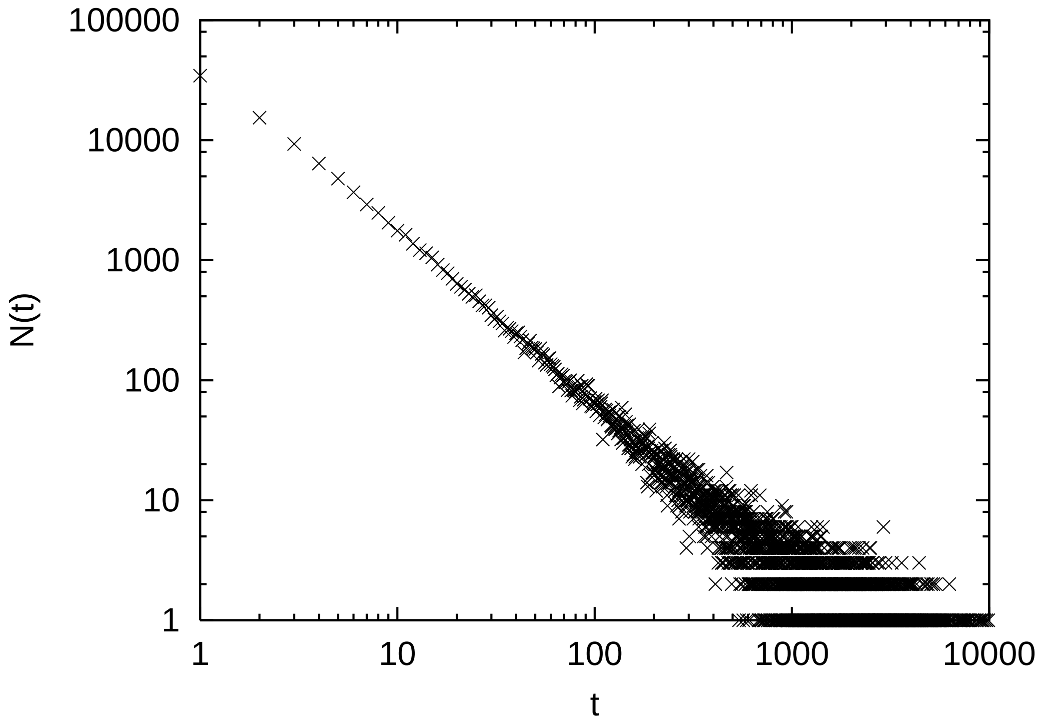


Fig. 2. Distribution of bull (bear) market durations, defined as phases with $M(t) > 0$ ($M(t) < 0$).

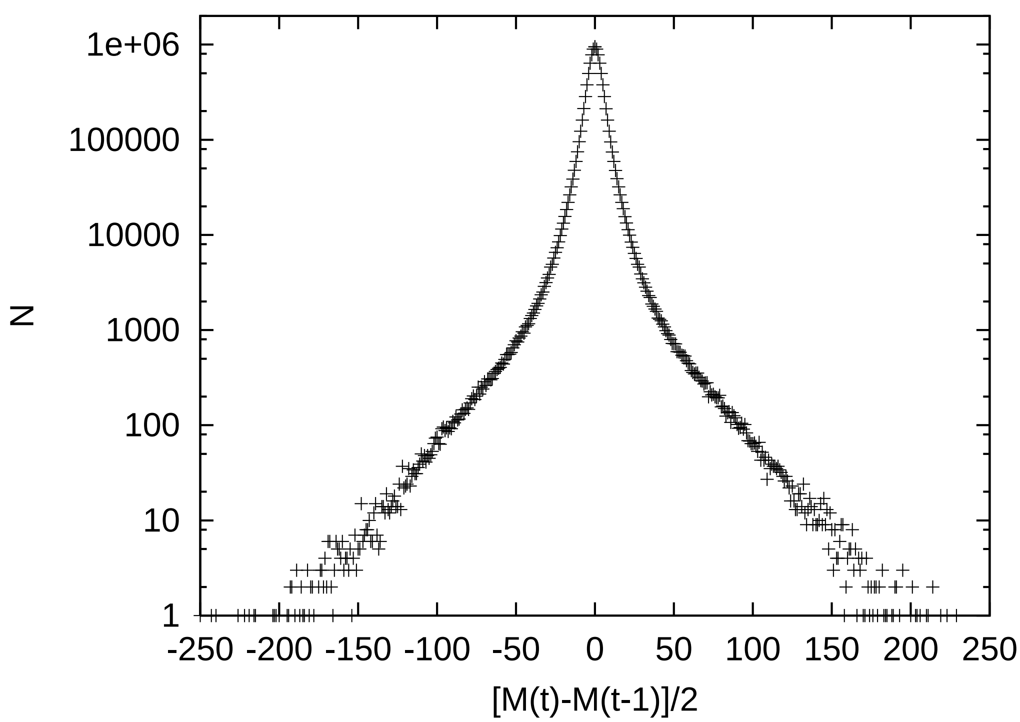


Fig. 3. Fat tails of the distribution of log-returns.

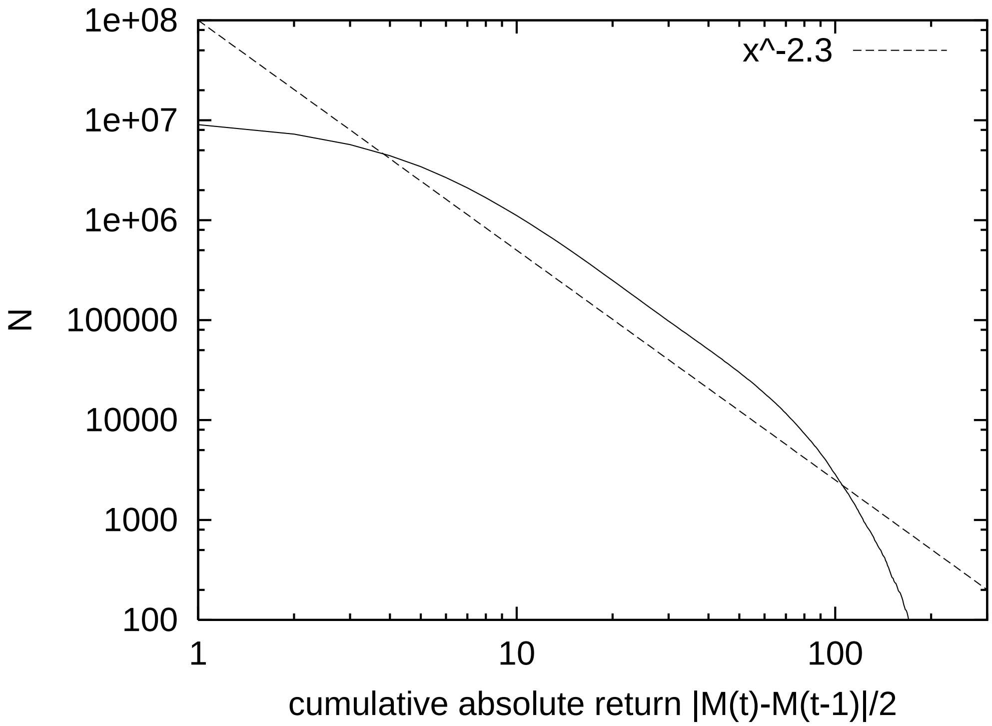


Fig. 4. Cumulative distribution of log-returns.

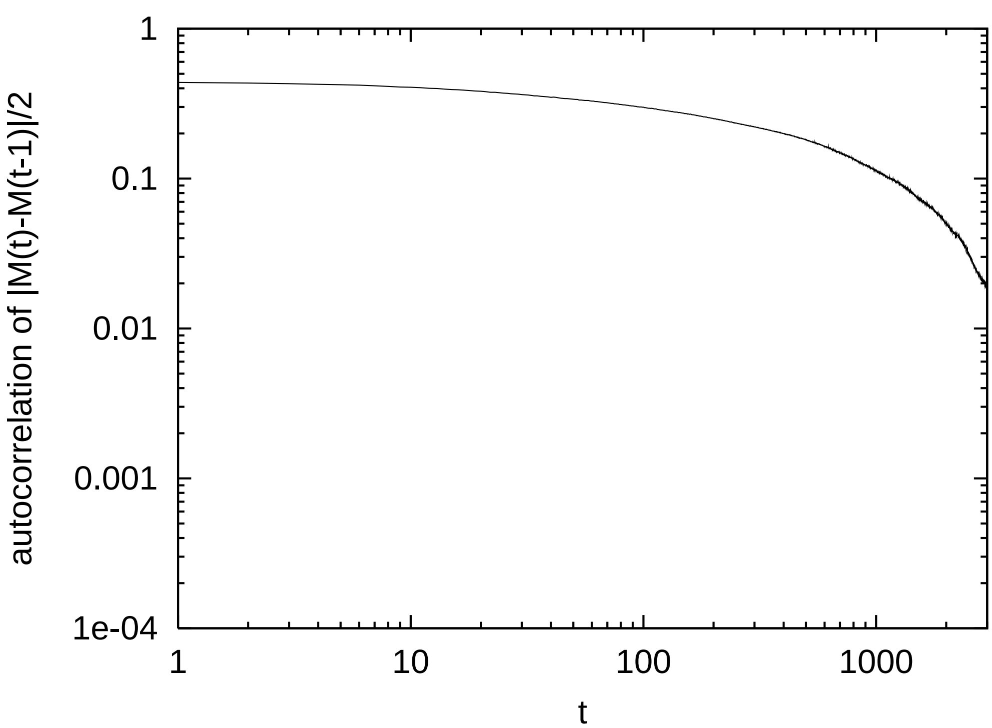
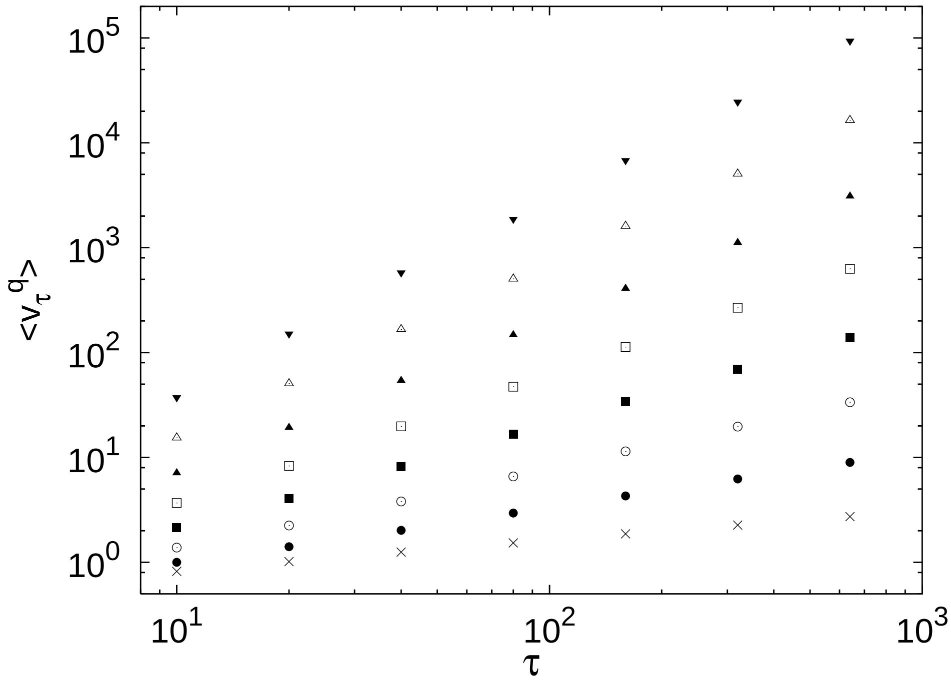


Fig. 5. Autocorrelation of absolute log-returns.

(a)



(b)

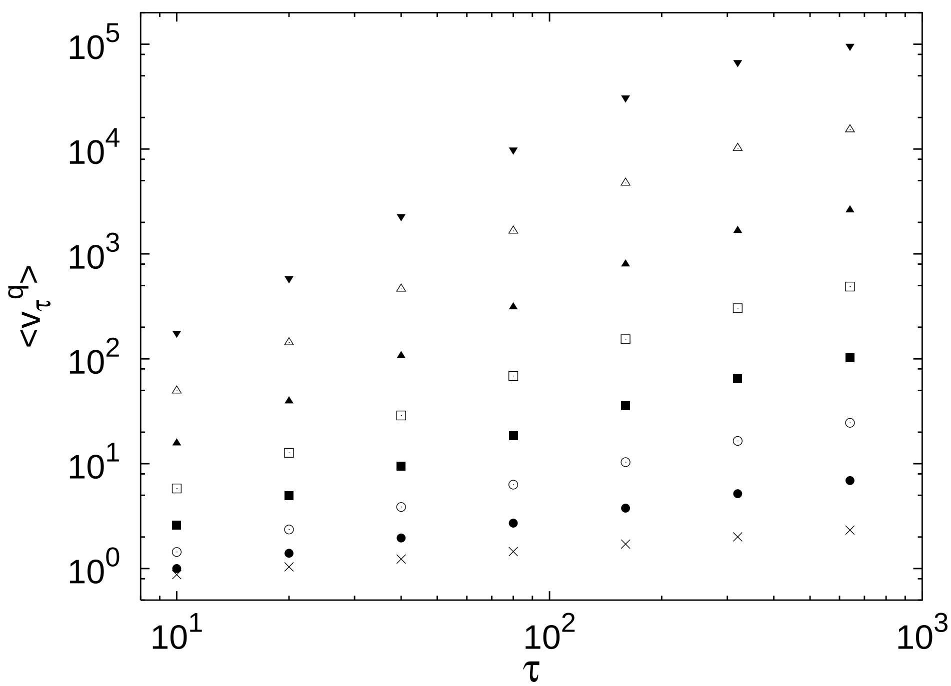


Fig. 6. Moments (q-th order) of volatilities v_τ at time-scales τ. (a) For a stock in NYSE with τ in minutes. (b) For the simulation result with τ in time-steps. In both plots, q ranges from 0.5 (cross) to 4.0 (triangles downward) with increment 0.5.